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COMMENT

Comment on 'Quantum mechanics of smeared particles'

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Abstract

In a recent article, Sastry has proposed a quantum mechanics of smeared particles. We show that the effects induced by the modification of the Heisenberg algebra, proposed to take into account the delocalization of a particle defined via its Compton wavelength, are important enough to be excluded experimentally.

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The idea to represent a particle not as an idealized point particle but instead as a *smeared* particle is not new, but recently a subtle way to introduce this smearing has been proposed. This formalism, used in [1] with some modifications, was first developed by Kempf *et al* [2]. The idea is to modify the commutation relations between position and momentum, (the Heisenberg algebra) to introduce a new short-distance structure characterized by a finite minimal uncertainty Δx_0 in position measurements. The existence of this minimal observable length is suggested by quantum gravity and string theory [3–7]. In this context, the new short-distance behaviour would arise at the Planck scale, and Δx_0 would correspond to a fundamental quantity closely linked with the structure of spacetime [8]. Kempf has suggested that this formalism could also be used to describe, as an effective theory, non-pointlike particles such as hadrons, quasi-particles or collective excitations [9]. In this case, Δx_0 is interpreted as a parameter linked with the structure of these particles and their finite size; no attempt is made to give an explicit link between this parameter and some fundamental property of the particle: it is considered as a free parameter.

In a recent article, Sastry has suggested that the deformation parameter of the Heisenberg algebra is given by the Compton wavelength of the particle [1]. He pointed out that in the case of the hydrogen atom, and in general in the quantum theory of atoms, the quantum mechanics of point particles gives an accurate description because the characteristic size of the smearing of the electron (the Compton wavelength) is α (the fine structure constant) times smaller than the characteristic size of the atom a_0 (the Bohr radius). Even if this assertion is correct the effects of this smearing of the electron are still too large and can be excluded by comparison between *standard* theoretical calculation and experimental data.

The modification of the energy level positions of the hydrogen atom introduced by the use of the new commutation relations between position and momentum has been evaluated to first order in [10]. The order of magnitude of the correction is given by $(\Delta x_0)^2 m^3 \alpha^4$, where m is the mass of the electron ($\hbar = c = 1$). The use of the Compton wavelength as the deformation parameter of the Heisenberg algebra, $\Delta x_0 \propto 1/m$, leads to a correction of the same order (10^{-3} eV) as the first relativistic kinematic and the spin–orbit corrections (which describe the fine structure of the hydrogen energy levels) and is thus two orders of magnitude larger than the Lamb shift and hyperfine structure corrections (see, for example, [11]). The agreement between *standard* theory and experiment is about 1 MHz (10^{-8} eV) for the Lamb shift of the 1S state [12, 13] and about 0.1 MHz for the hyperfine structure of the 1S state (the famous 21 cm hyperfine transition) [14, 15]. This excellent agreement definitely excludes the proposal of Sastry.

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